

On the Transformation of Thermodynamic Quantities in the Special Theory of Relativity

Martin Gubler, Kantonsschule Frauenfeld, gub@stafag.ch

Amazingly, the question of how temperature is transformed in the special theory of relativity is still controversial in the literature: What is the value T' we should ascribe to a relativistic observer of the temperature of a quantity of gas when it has the value T in the rest frame? Is it even possible to ascribe consistent transformations for the thermodynamic variables? One of the first to take up this question was none other than Max Planck with his doctoral candidates Kurd von Mosengeil and Max von Laue.

We will cautiously assume that such transformations do exist and show how little wiggle room there is for these transformations, given a few elementary results of the special theory of relativity. Within this narrow scope, we opt for one of two possible solutions which will actually amount to a deeper definition of the concept of temperature. We justify the choice in detail and then show that the second choice would have led to exactly the same results as for Planck et al. The proposals of many other authors however, including such famous persons as Eddington, are not logically consistent.

We restrict our considerations to temperatures from 0 K to about 100 million K. In this range gas particles in the rest frame have velocities that are at most a small percent of c , and we can still use the standard formula to obtain a very good approximation of the kinetic energy of the particles. At much higher temperatures some of our formulas are useless. We console ourselves with the fact that the temperature at the center of the sun is only about 15.6 million K ...

Translated to the Latin of our days by Rick Sermersheim in July 2013, version 1.02 of December 2013

1 Thermodynamic State Variables

Individual particles of a gas have the 'microscopic' properties velocity and momentum. The entire gas, however, is attributed the 'macroscopic' properties volume, pressure, density, temperature, internal energy, enthalpy and entropy. Moreover, the gas consists of a specific number of particles, which can be written as a multiple of the Avogadro constant.

These macroscopic state variables are constant when the gas is in thermal equilibrium. They are path independent, in that they do not allow any conclusions to be drawn as to how the system got to equilibrium (in phase space). We also note that the state variables are scalars.

Process variables such as the work done through expansion or the amount of heat absorbed or emitted, however, are strongly dependent on the path of change.

We use the usual variables for all these values. Non-prime capital letters denote values that are measured in the rest frame, i.e. from an observer who is not moving relative to the center of mass of the gas. With primed capital letters we denote the corresponding values that 'fast-moving observers' would ascribe to the gas.

2 The Transformations of These State Variables

We assume that there *exist* transformations for each scalar state variable as well as for each process variable, and that these transformations are given by a multiplicative factor whose value depends only on relative speed:

$$X' = f_X(\vec{v}) \cdot X \quad \text{where} \quad f_X(0) = 1$$

A priori it is not certain that a set of such transformations can be found! For example take the pressure of a gas. A fast-moving observer could measure a different pressure in the direction of his velocity than in a direction perpendicular to it. Would that not explain the deformation of the gas balloon to an ellipsoid of revolution? The at-rest spherical balloon is indeed subjected to the Lorentz contraction.

In fact, the change of momentum of a given particle bouncing against the balloon wall must be calculated for all directions *relative to the balloon wall*, and therefore the considerations in section 6 for the transformation of the pressure are direction independent. That the balloon loses its spherical shape has more to do with how one must measure lengths of fast-moving objects. Ultimately this can be traced back to the different observer's assessment of simultaneity. The fast-moving observer easily determines that the balloon must have a spherical shape in the rest frame, even when it presents itself as somewhat flattened.

3 States of Equilibrium

We derive all the transformations based on the observation of a given quantity of an ideal gas, which is enclosed in a (variable) volume V . If such transformations exist, which are applicable to all systems, then they have to be applicable to an enclosed mole of an ideal gas too !

If such a gas is thermally isolated (i.e., it neither absorbs nor radiates energy to its environment), then for statistical reasons it will very quickly reach an equilibrium state. It is essential that this statement is also true from the perspective of a fast-moving observer:

If, for example, pressure, volume and temperature are constant in the rest frame, then the same will hold for P' , V' and T' . Furthermore, this is true regardless of the exact formulation of the corresponding transformations. The relative velocity is indeed constant as assumed in the SRT, and therefore P' has to be constant too:

$$(P \text{ constant and } \vec{v} \text{ constant}) \Rightarrow P' = f_p(\vec{v}) \cdot P \text{ constant}$$

But if P' , V' and T' are likewise constant, this means nothing other than that the gas is also in a state of equilibrium for the fast-moving observer. Therefore, the presence of a state of equilibrium is not 'relative'. It is a fact, given for all frames of inertia - or for none.

4 The Transformation of Lengths and Volumes

It is well accepted that the following formula is used to transform volumes:

$$V' = V \cdot \sqrt{1 - \frac{v^2}{c^2}}$$

As we have already noted, prime values denote what an observer measures who moves past the object under consideration with relative velocity v in the x -direction, while the unprimed values denote an observer at rest, relative to the observed system. The unprimed values are thus identical to the rest values. The volume transformation results from the 'length contraction' in which only the direction of the relative movement is affected. Shape and orientation of the volume do not matter.

Because this radical expression occurs repeatedly we will often just refer to 'the root' or the 'root factor' and write $V' = V \cdot \sqrt{\quad}$ for the transformation of the volume.

5 The Transformation of Particle Density

The Avogadro constant N_A defines the amount of particles in a mole. It describes a certain number of atoms or molecules, and it is directly related to the rest mass of the amount of a substance. This rest mass and the Avogadro constant are relativistically invariant.

Generally, absolute numbers such as the number N of gas molecules enclosed in a vessel are independent of the relative speed of an observer. But since volumes have to be transformed the particle density, i.e. the number of particles per unit volume, is not invariant. For an observer flying past, the density increases, since the same number of particles is compressed into a smaller volume. The root factor is in the denominator. We refrain from introducing a symbol for the particle density.

6 The Transformation of Pressure

Pressure is a scalar quantity. In a gas or in a liquid it is (in the absence of a gravitational field) the same everywhere. It is isotropic, that is, it generates in all directions the same force per square centimeter on the walls of the enclosing vessel.

Therefore we can concentrate on the motion of the particles in the y-direction against the vessel's wall as the vessel is moving towards us in the x-direction with relative velocity v . Momentum in the y-direction transforms according to the formula $p_y' = p_y$. The movement is slowed in the transverse direction by the root factor, however this is (according to traditional terminology) compensated by the relativistic increase in the mass of the particle.

Because the transverse velocity u' of the particle in the y-direction appears slower to us it should follow that per unit time fewer particles will strike the vessel wall - if the particle density were unchanged. But the particle density increases due to the length contraction by exactly the same factor as the transverse velocity is decreased. So overall the same number of particles per unit time encounter the vessel wall with the same momentum, generating the same pressure that an observer would measure in the rest frame. The pressure P therefore transforms as follows:

$$P' = P$$

So far, almost all authors agree ...

7 Ideal Gas and the Definition of Temperature

Even with a primitive thermometer one can very accurately check if temperature is constant or not. For arbitrary gases or gas mixtures the following is true with high accuracy:

$$P_1 \cdot V_1 = P_2 \cdot V_2 \quad \text{if } T = \text{constant}$$

Amazingly, experiments further show that all gases and gas mixtures have the same coefficient of volume expansion! Extrapolation leads to absolute zero at -273.15°C and by using a clever translation of the temperature scale, we obtain the simple relation

$$V_1 / T_1 = V_2 / T_2 \quad \text{if } P = \text{constant}$$

Putting these two insights together, we get the following equation for any two equilibrium states of the same amount of gas:

$$\frac{P_1 \cdot V_1}{T_1} = \frac{P_2 \cdot V_2}{T_2}$$

Clausius and Boltzmann have shown with the model of the *Ideal Gas*, that this equation holds exactly when the gas particles are much smaller than their average distance travelled between two collisions, and if the particles can be regarded as perfectly elastic colliding balls, which exercise no forces on each other at distance. Finally these insights provide a definition of temperature: Take an ideal gas (helium is used in practice) and measure its temperature using

$$T = P \cdot V \cdot T_0 / (P_0 \cdot V_0)$$

where the reference values T_0 , V_0 and P_0 can be freely chosen. A helium gas thermometer as a primary thermometer further eliminates the need to measure volumes: The volume is kept constant through pressure adjustment, and so we get

$$T = T_0 \cdot P / P_0 \quad \text{if } V = \text{constant}$$

The triple point of water is taken as the reference temperature T_0 . It is assigned the value of 273.160 K by definition. The measurement of temperature is thus reduced to a pressure measurement.

The Physikalisch-Technische Bundesanstalt in Braunschweig, Germany is one of very few metrological organizations worldwide that operate such a helium-gas thermometer as a primary-thermometer. Reference temperatures for triple points and melting points of all sorts of pure materials are measured. These reference temperatures are easily reproducible in every laboratory worldwide, and thus calibration curves for simpler thermometers (for example, resistance thermometers or mercury thermometers) can be determined.

The temperature scale is thus *defined* by the gas law, and this definition can be *realized* with a gas thermometer in the range of approximately -150° C to 1500° C which is, metrologically, of critical importance. At higher temperatures, temperature measurement relies on the laws of radiation, for low temperatures reference values are negotiated at conferences, which are periodically refined. At low temperatures, for example, thermal noise is used to implement a primary thermometer. Another primary method is based on measuring the speed of sound in gas.

Much information on this subject is provided by the website of the Physikalisch-Technische Bundesanstalt in Braunschweig, Germany. The link is <http://www.ptb.de/cms/fachabteilungen/abt7/fb-74.html>.

Defining a fundamental unit always means two things: One must first establish a unit of measurement, and secondly provide a practical procedure for doing the measurement. Measurement of time provides the best example. The definition of the second can hardly be separated from the *realization* of this second through arrays of atomic clocks. The second is so to speak industrially manufactured nowadays ...

Metrologically, it is *impossible* to define temperature via the average energy of particles, via a change in entropy or via heat quantities; since no one knows how to measure these quantities with reasonable accuracy. Temperature is defined as a macroscopic quantity *before* it is associated with microscopic terms and long before one can estimate changes in total entropy. Temperature is - like time and mass - a fundamental unit of physics. This does not mean, however, that it has to be relativistically invariant.

Furthermore, the concept of temperature gives time its direction in physics! Thermal equilibrium is reached when two bodies are at the same temperature, and two bodies in thermal contact always strive to reach the same temperature. The concepts of entropy and heat are not needed to introduce the famous "arrow of time" - temperature is sufficient.

8 The Transformation of $k \cdot T$ and $R \cdot T$

If one changes one or more of the state parameters of a specific amount of enclosed helium or neon at temperatures above -100°C , then a new equilibrium soon arises with new values for pressure, volume and temperature:

$$\frac{P_1 \cdot V_1}{T_1} = \frac{P_2 \cdot V_2}{T_2}$$

According to Section 3, a fast-moving observer sees two equilibrium states of the same amount of substance too, and from Section 2 and the equation above we get

$$\frac{f_P(\vec{v}) \cdot P_1 \cdot f_V(\vec{v}) \cdot V_1}{f_T(\vec{v}) \cdot T_1} = \frac{f_P(\vec{v}) \cdot P_2 \cdot f_V(\vec{v}) \cdot V_2}{f_T(\vec{v}) \cdot T_2}, \text{ also } \frac{P_1' \cdot V_1'}{T_1'} = \frac{P_2' \cdot V_2'}{T_2'}$$

regardless of the details of the individual transformations! The gas equation is therefore form-invariant.

This equation is also satisfied if we set P_2 to the standard pressure $P_0 = 101'325 \text{ Pa}$, set V_2 to n times the standard molar volume of $V_0 = 0.022'414 \text{ m}^3/\text{mol}$ and set T_2 to the standard temperature $T_0 = 273.15 \text{ K}$. We denote, as usual, the amount of substance in moles with $n = N / N_A$ and reduce the constants on the right side, obtaining

$$\frac{P_1 \cdot V_1}{T_1} = n \cdot \frac{P_0 \cdot V_0}{T_0} = n \cdot R = N \cdot k$$

This is the *definition* of the universal gas constant R and the Boltzmann constant k ! R is the product of a pressure and a volume, divided by a temperature. So R has to transform accordingly: Following 4 and 6 R and k can only be invariant when temperatures transform just as volumes do.

Thus the following applies to a certain quantity of a gas in all states of equilibrium

$$P \cdot V = n \cdot R \cdot T = N \cdot k \cdot T$$

Even in this notation, the gas equation must be form-invariant. R is only the abbreviation for $P_0 \cdot V_0 / T_0$. But we already know how to transform the left side of this equation for a fast-moving observer:

$$P' \cdot V' = P \cdot V \cdot \sqrt{\quad}$$

Since the particle number N , as well as the amount of substance n are invariant it follows for the transformation of k and T that

$$k' \cdot T' = k \cdot T \cdot \sqrt{\quad}$$

The product of k and T (as well as the product of R and T) are transformed through multiplication with the root. We will now derive the same result by a different route.

9 The Transformation of Average Translational Energy

Simple statistical considerations show that the pressure that an ideal gas exerts on a vessel wall is proportional to the average particle density and can be calculated using the following formula:

$$P = \frac{1}{3} \cdot \frac{N}{V} \cdot m \cdot \overline{u^2}$$

where the velocity of a gas particle is measured relative to the center of mass of the entire gas. If we consider again (as in Section 6), the y-direction of the relative motion, we know exactly how the individual terms of this equation behave. We can thereby avoid introducing the concept of force: The pressure is equal to the average change in momentum that need be applied per unit area of the vessel and per time unit.

With $P' = P$, $N' = N$, $V' = V \cdot \sqrt{1 - \frac{v^2}{c^2}}$ and $m' = m / \sqrt{1 - \frac{v^2}{c^2}}$ it follows from the above formula

$$\left(\overline{u^2}\right)' = \overline{u^2} \cdot \left(1 - \frac{v^2}{c^2}\right)$$

For particle velocities u or u' , which are small compared to the speed of light, we may use the classical formula for kinetic energy. Hence we get

$$P = \frac{1}{3} \cdot \frac{N}{V} \cdot m \cdot \overline{u^2} = \frac{2}{3} \cdot \frac{N}{V} \cdot \overline{E_{kin}}$$

This equation applies *mutatis mutandis* for a fast-moving observer, since the average kinetic energy transforms like the transverse velocity u :

$$\left(\overline{E_{kin}}\right)' = \frac{1}{2} \cdot m' \cdot \left(\overline{u^2}\right)' = \frac{1}{2} \cdot \frac{m}{\sqrt{1 - \frac{v^2}{c^2}}} \cdot \overline{u^2} \cdot \sqrt{1 - \frac{v^2}{c^2}} = \overline{E_{kin}} \cdot \sqrt{1 - \frac{v^2}{c^2}}$$

We would have obtained this result immediately if we had replaced all velocities u in the y-direction directly with $u' = u \cdot \sqrt{1 - \frac{v^2}{c^2}}$.

Thus the average kinetic energy of the particles and V transform by multiplying by the root factor. The value of the term on the right side in the second to last formula remains unchanged, as does the value of the pressure on the left side.

10 The Transformation of $k \cdot T$ and the Average Translational Energy

Multiplying the equation in the middle of this page by V , we obtain

$$\frac{2}{3} \cdot N \cdot \overline{E_{kin}} = P \cdot V = N \cdot k \cdot T \quad \text{und daraus} \quad \overline{E_{kin}} = \frac{3}{2} \cdot k \cdot T$$

The product $k \cdot T$ clearly must transform in the same manner as the average kinetic energy, that is, by multiplying by the root factor. In the following sections, we will derive transformation formulas for other quantities, without knowing the individual transformations of k and T .

11 The Transformation of Internal Energy and Heat Quantities

For an ideal atomic gas the internal energy U is nothing other than the kinetic energy of (point-like) gas particles. By adding a certain amount of heat energy in an isochoric process we get

$$\Delta Q = \Delta U = N \cdot \overline{\Delta E_{kin}} = N \cdot \frac{3}{2} \cdot k \cdot \Delta T = n \cdot \frac{3}{2} \cdot R \cdot \Delta T$$

and it is clear that the internal energy U and a portion of heat energy need to transform like the average kinetic energy, or the product of k and T , that is, by multiplying by the root factor. Incidentally, we see here that the molar heat capacity C_V must transform like R .

12 The Transformation of Enthalpy

The enthalpy H of an amount of material is composed of the internal energy U and the so called thermodynamic work $P \cdot V$, which is required to 'shovel free' the volume V for the given amount of material under an external pressure P . According to previous results, the enthalpy also transforms by multiplying by the root factor:

$$H = U + P \cdot V \quad \text{implies} \quad H' = H \cdot \sqrt{}$$

P is invariant, and U and V transform through multiplication by the root.

13 The First Law of Thermodynamics

At least in the isobaric expansion of an ideal gas, we can check the invariance of the first law, given that the work can continue to be calculated as the product of pressure and increased volume:

$$\Delta U = \Delta Q' - P \cdot \Delta V$$

All summands on the right side transform through multiplication by the root. Even before we choose a specific transformation for T it is clear that the first law of thermodynamics is 'saved' by assuming that all energy exchanges, especially the work through expansion, transform in the same way as changes to internal energy or heat quantities. This is just the assumption that energy conservation should not be violated even for a fast-moving observer.

Explicitly, we can check this assumption by the following calculation:

$$\Delta W' \cdot \sqrt{} = \left(\int P(V) \cdot dV \right) \cdot \sqrt{} = \int P'(V') \cdot (dV \cdot \sqrt{}) = \int P'(V') \cdot dV' = \Delta W' \cdot \sqrt{}$$

14 The Second Law of Thermodynamics

The second law in the formulation of the maximum efficiency of a heat engine also remains unaffected. The expression

$$\eta = \frac{T_1 - T_2}{T_1}$$

is invariant, no matter with which factor the temperatures are transformed. The upper bound for the efficiency of a heat engine does not change for a fast-moving observer. Also for him it is impossible to build a 'perpetual motion machine of the second kind' ...

15 Interim Conclusions

According to the previous considerations, the following quantities are invariant in the sense of STR:

- pressure P (and pressure changes)
- particle number N and amount of substance $n = N / N_A$
- thermodynamic efficiency as a quotient of temperatures

The following quantities are transformed by multiplying by the root factor:

- volume V (and volume changes)
- the products $k \cdot T$ and $R \cdot T$
- the average velocity and the root of the average squared velocity of the particles
- the average energy of individual particles
- heat quantities Q , internal energy U , performed work W (and their differences)
- enthalpies H (and their differences)

The following laws continue to apply *mutatis mutandis* for all inertial observers:

- the first law of thermodynamics
- the second law of thermodynamics
- the ideal gas law

$$p \cdot V = n \cdot R \cdot T = N \cdot k \cdot T$$

- for non-relativistic ideal gases:

$$\overline{E_{kin}} = \frac{3}{2} \cdot k \cdot T$$

- for non-relativistic ideal gases:

$$P = \frac{1}{3} \cdot \frac{N}{V} \cdot m \cdot \overline{u^2} = \frac{2}{3} \cdot \frac{N}{V} \cdot \overline{E_{kin}}$$

16 The Transformation of k and T

From SRT and thermodynamics it follows (see 8 and 10) that

$$k' \cdot T' = k \cdot T \cdot \sqrt{}$$

But how should we transform T and k individually?

At this point, there is a parting of expert opinion. A majority assumes without further ado that the Boltzmann constant k should be invariant. Many of these authors have most likely rashly assumed that the entropy S , being a probability, must be invariant. This is not necessarily true, as we will show in the following sections.

Only a few authors take the alternative easy route and bank on the temperature T being invariant. It follows that k and R must transform and hence entropy is not an invariant quantity. But this is not tragic and the validity of the second law is not jeopardized. We will join this small contingent, which was first established in 2003 by I. Avramov.

But there are also authors who hold the opinion that both k and T are invariant. Others claim that the temperature should increase with increasing relative velocity, while k and S are invariant. There is almost no combination of assertions which is not represented by someone in a recent publication! It is this cacophony of opinions in publications of the past 20 years, which has led me to investigate the matter myself.

And actually it might be even worse: both k and T could transform by multiplying by the root of the root factor: The product would then once again satisfy the above requirement! Many other decompositions of the root term into 2 factors are conceivable. We do not want to further illuminate this abyss, since actually there is much in favor and nothing against our setting the crown of invariance on the temperature T . An argument follows in section 22.

But first we want to take a closer look at the concept of *entropy*, so that we can assess the damage we cause, if we apply $k' = k \cdot \sqrt{}$. Secondary school students are often not familiar with entropy, so we need to dig a little deeper.



Um die unsichtbare Dunkle Materie, die unsichtbare Dunkle Energie, die unsichtbaren Extradimensionen sowie die unsichtbaren Blasenuniversen zu erklären, habe ich eine unsichtbare Gleichung hergeleitet!

"In order to explain the invisible dark matter, the invisible dark energy, the invisible extra dimensions and the invisible parallel universes, I have derived an invisible equation!"

17 The Transformation of Entropy in Information Theory

In information theory the *size of ignorance or uncertainty* in a certain amount of information is measured by determining the number of binary questions one would ask in order to know everything. For example, consider that a modem has received 1 character in 8-bit ASCII code. 256 different characters are possible. If all characters occur with the same probability, 8 yes-no questions are necessary on average to identify the character. So the entropy of this information is

$$S_{inf} = \log_2(256) = 8$$

If one knows in addition that the most significant bit is 0, the entropy decreases immediately to 7, because it must be a character in the range of 0 to 127. And if we know that the character 'A' = 65 (dez) = 0100'0001 (bin) has been received, the entropy decreases to $0 = \log_2(1)$.

Assuming that all cases are equally likely, entropy measures the number of possibilities given our knowledge of a particular situation that still come into question. The measure is taken as a base 2 logarithm. The entropy is thus a quantitative indication of our 'ignorance' or 'uncertainty' or our lack of information.

The fundamental unit of entropy in information theory is the bit, where 8 bits are grouped together to form a byte. Physically, the entropy of the information is unitless. The unit 'bit' has a function similar to that which 'radian' has to angle measurement. A hard disk of size 2 *tebibyte* (knowing nothing about its content) demonstrates an entropy of size $\log_2(2 \cdot 2^8 \cdot 2^{40}) = 49$ (with a 2-*terabyte* hard drive, it is $\log_2(2 \cdot 2^8 \cdot 10^{12}) \approx 48.863 \dots$). Thanks to the logarithmic function, the entropy does not increase astronomically even with huge amounts of information.

If I have left my car key in one of 8 school rooms I visited today, the entropy is $\log_2(8) = 3$. If I also forgot my hat in one of these places, the number of possible situations increases to $8 \cdot 8 = 64$ and the entropy increases to 6. The entropy of N objects that can be located in m places *independently*, is $N \cdot \log_2(m)$. Entropies simply add. However, this does not apply to the house key on my keychain, because I have not left it independently of the car key, since it is attached to the same key chain.

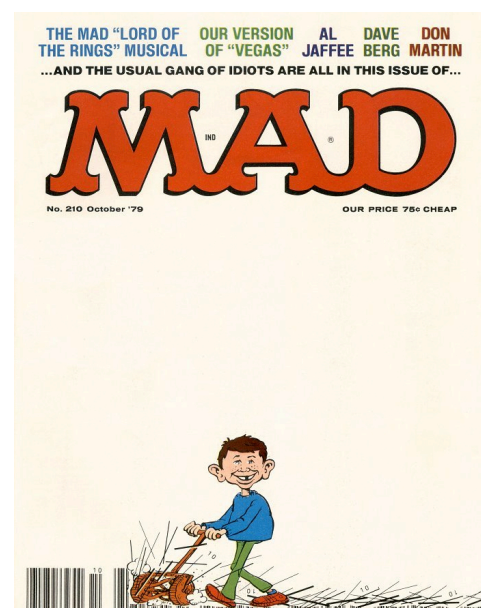
The macro-state "my car key is in one of the rooms I visited today at school" is my current state of knowledge. It includes 8 possible micro-states corresponding to a maximum level of knowledge. The entropy of the macro-state is the logarithm of the number of micro-states, which belong to this macro-state.

We now have much of the terminology in place for the different kinds of entropy pertinent to physics. It should also be clear that the entropy of the information does not change when one examines from a fast-moving spacecraft either the school house with the misplaced hat or the aforementioned 2-terabyte hard drive. The number of micro-states is independent of the relative movement. So the following transformation formula holds for the entropy of information:

$$S_{inf}' = S_{inf}$$

A bar code still contains the same amount of information when subjected to a length contraction in whichever direction. This reminds me of a MAD cover illustration of the 70s: Alfred E. Neumann lowering the price by giving a barcode a 'trim' with a lawn mower ...

It's a Mad world on the internet: Rick found the corresponding cover page on the web. Visit madcoversite.com



18 The Transformation of the Entropy of Volumes

A gas particle is somewhere in a volume V_1 . We turn a dial doubling the volume where the particle can be. According to the ideas of the last section, we are also doubling the number of micro-states available to the particle. It doesn't matter what elementary unit volume V_0 we assign to a micro-state, the entropy increases, and we have

$$\Delta S_{\text{inf}} = \log_2 \left(\frac{\frac{V_2}{V_0}}{\frac{V_1}{V_0}} \right) = \log_2 \left(\frac{V_2}{V_1} \right) = \log_2(2) = 1$$

In physics the *natural logarithm* is used to measure the entropy. But this differs only by a constant factor from the base 2 logarithm. Much more important is the fact, that in the context of physics the logarithm is multiplied with the Boltzmann constant k . Thus our above change of volume for 1 particle has the measure

$$\Delta S_{\text{vol}} = k \cdot \ln \left(\frac{V_2}{V_1} \right) = k \cdot \frac{1}{\log_2(e)} \cdot \log_2 \left(\frac{V_2}{V_1} \right) = k \cdot \frac{1}{\log_2(e)} \cdot \Delta S_{\text{inf}} \approx 0.693 \cdot k \cdot \Delta S_{\text{inf}}$$

By multiplying by k , the entropy artificially, so to speak, obtains the units of k , i.e. J/K. In the next section we will see the advantages of this definition. This new scaling changes nothing at all for the present. However, now the entropy is invariant if and only if k is !

If one has N particles instead of one, each residing independently somewhere in the volume, then a change in volume from V_1 to V_2 causes a corresponding change in the volume entropy of

$$\Delta S_{\text{vol}} = N \cdot k \cdot \ln \left(\frac{V_2}{V_1} \right)$$

By whatever factor the volume transforms for a moving observer the volume entropy must transform the same as the Boltzmann constant k !

We would obtain very similar results with the concentration entropy S_K , the entropy of mixing S_M , free entropy S_F or spin entropy S_S .

All these changes in entropy equal the product of k and the logarithm of a ratio of micro-states, and the number of the micro-states do not change due to a relative movement of the observer. They are themselves calculated as a ratio of two 'volumes' (maybe also volumes in phase space). It is irrelevant how exactly the numerator and denominator transform, because the corresponding factor cancels out.

However, the calculation of the total entropy of a given amount of an ideal gas is by no means trivial, since the various micro-states, for example of kinetic energy of a gas particle, may assume very different probabilities. It is not just about counting the number of micro-states, since these are not evenly distributed.

The corresponding calculations can be found in the literature. For secondary school, the book "Entropie und Information - naturwissenschaftliche Schlüsselbegriffe" by Wolfgang Salm (Aulis Verlag 2002) is recommended. The sections 17 to 20 of this work are based largely on that publication.

19 The Transformation of the Entropy of the Internal Energy U

If for an amount of material comprising $N = n \cdot N_A$ particles at temperature T , a small amount ΔQ of energy is added, then the temperature of the material will increase by ΔT . The corresponding entropy change is given by

$$\Delta S_Q = \frac{1}{2} \cdot f \cdot N \cdot k \cdot \frac{\Delta T}{T}$$

where f stands for the number of degrees of freedom. Again we immediately see from the expression that it must transform as k does, regardless by which factor temperatures are transformed.

In an ideal gas, the (point-like) particles have only the three translational degrees of freedom of movement. When heated isochoric, this results in the following formula:

$$\Delta S_Q = \frac{3}{2} \cdot N \cdot k \cdot \frac{\Delta T}{T} = n \cdot \frac{3}{2} \cdot N_A \cdot k \cdot \frac{\Delta T}{T} = n \cdot \frac{3}{2} \cdot R \cdot \frac{\Delta T}{T} = n \cdot \frac{C_V \cdot \Delta T}{T} = n \cdot \frac{\Delta Q_V}{T}$$

But even with further internal degrees of freedom, the ratio $\Delta T/T$ is the fundamental measure of changes in entropy; all temperature-independent factors cancel out as constants in the first derivative of the logarithmic function. For solids at room temperature we have $f = 6$, the atoms are in a lattice structure and have three rotational degrees of freedom and three degrees of vibrational freedom. We obtain

$$\Delta S_Q = \frac{6}{2} \cdot N \cdot k \cdot \frac{\Delta T}{T} = n \cdot 3 \cdot R \cdot \frac{\Delta T}{T} = n \cdot C_V \cdot \frac{\Delta T}{T} = n \cdot \frac{\Delta Q_V}{T}$$

All solids at room temperature should have the same molar heat $C_V = 3 \cdot R$. This is indeed the case. One speaks of the Dulong-Petit law. And it is clear that molar heat must transform the same as entropy and also the same as k and R .

If you place a disc of aluminum (10 moles, temperature 350K) on another disc of the same material (also 10 moles, but temperature 290K), the warmer plate will *always* impart energy to the cooler one, until the two discs have the same temperature of about 320K. The reverse process can never be observed, although conservation of energy would also apply here. The famous time arrow is thus disclosed. It is independent of the concept of entropy, but can be elegantly summarized: total entropy increases for all irreversible processes.

In our example, the entropy of the warmer plate decreases by approximately $10 \cdot 3 \cdot R \cdot 1/350$ as it cools from 350 K to 349 K. Meanwhile, the entropy of the cold plate increases to $10 \cdot 3 \cdot R \cdot 1/290$. Overall, this first step in the convergence of the temperatures gives an entropy increase of about $10 \cdot 3 \cdot R \cdot (1/290 - 1/350)$ J/K, or about 0.147 J/K.

The second law of thermodynamics can be formulated in various ways. One says that thermal energy only flows spontaneously from higher to lower temperatures. Another formulation is the following: processes occur spontaneously only so that the total entropy remains constant (reversible process) or increases. With the diffusion of a gas, the volume entropy increases and with heat exchange the heat entropy increases. For complicated processes, a partial entropy can also decrease. But the sum of all entropies, that is, the total entropy S always increases when the operation is not reversible.

20 The Transformation of the Total Entropy S According to Planck

Planck wrote in his 1907 paper ¹

"We want to prove that the entropy of the body under consideration in relation to the primed system has the same value as for the unprimed system. You could generally base this proof on the close relationship between entropy and probability, whose values cannot possibly depend on the choice of the reference system, however, here we prefer to take a more direct approach, independent from the introduction of the concept of probability.

We consider the body taken from a state in which it is at rest with respect to unprimed reference system and is brought by any reversible adiabatic process to a second state in which it is at rest for the primed reference system. If we denote the entropy of the body for the unprimed system in the initial state as S_1 , and that for the final state with S_2 , then because of reversibility and adiabaticity $S_1 = S_2$. But also for the primed reference system, the process is reversible and adiabatic, so we have also: $S'_1 = S'_2$.

Now if S'_1 is not equal to S_1 , but rather $S'_1 > S_1$ this would mean that: The entropy of the body in the reference system, which is undergoing motion, is greater than that for the reference system, in which it is at rest. Then, according to this proposition it must also be that $S_2 > S'_2$, for in the second case of the body is at rest in the primed reference system, and is considered to be in motion for the unprimed reference system. But these inequalities contradict the two equations established above. Nor can it be that $S'_1 < S_1$; consequently $S'_1 = S_1$, and in general:

$$S' = S$$

that is, the entropy of a body does not depend on the choice of the reference system."

Is Planck's argument watertight? Even the savvy Pauli accepts it in his famous encyclopedia article ² on relativity theory (§46, p.694). However, it suffers from a mistake in logic called *petitio principii* (i.e., begging the question): what one wants to show is already used in the proof.

Planck must bring a body initially at rest (first state) in the unprimed system to the velocity v of the primed system so that in this second state it is at rest in that system. So in the unprimed system one already measures the entropy S_2 on the fast-moving body, while reciprocally in the primed system one measures S'_1 on a fast-moving body. Planck simply assumes that S_2 will have the same value as S_1 , but this is exactly what would have to be shown! It could also be that $S_2 = S_1 \cdot \sqrt{\dots}$, as it would precisely be, if entropies of fast-moving systems transform! This would then give the following, also contradiction-free set of measurements:

$$S_2 = S_1 \cdot \sqrt{\dots}, \quad S'_2 = S_1, \quad S'_1 = S_2 \cdot \sqrt{\dots}, \quad S_2 = S'_1$$

For reasons of symmetry, the two formulas without root factor must apply. The set of all four formulas is consistent with or without the root factor!

So Planck simply *assumes* that entropy is invariant. From this 'result', he then correctly concludes that the transformation of the temperature is $T' = T \cdot \sqrt{\dots}$.

In section 22 we will present the argument against Planck, Einstein and Pauli, along the lines of Avramov. But first we again consider the second law of thermodynamics, this time in the formulation of entropy.

1 http://wikilivres.ca/wiki/Zur_Dynamik_bewegter_Systeme

2 <http://gdz.sub.uni-goettingen.de>

21 The Second Law of Thermodynamics and the Transformation of Entropy

Of course Planck is right when he says that entropy is closely related to a probability. Strictly speaking, it is the product of k with the logarithm of the inverse of a probability:

$$S = k \cdot \ln(\Omega) \quad (\Omega \text{ stands for the inverse of an invariant probability})$$

The entropy S must therefore transform the same as k . Therefore, the relationship

$$dS \cdot T = dQ$$

still holds, as the product $S \cdot T$ transforms like $k \cdot T$, i.e., by multiplying by the root factor.

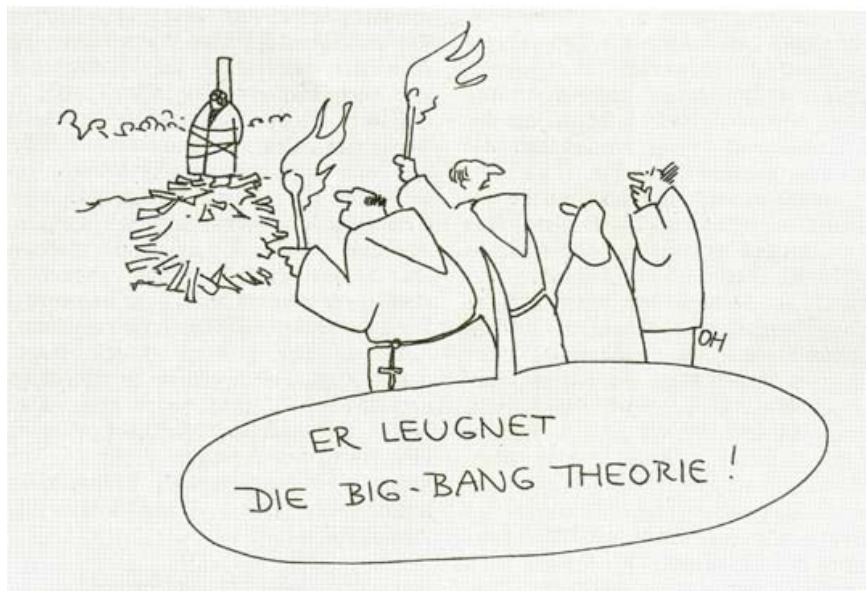
In section 14, we have already seen that the second law of thermodynamics remains valid also for a fast-moving observer. On this point, all authors are for once agreed. The STR respects the arrow of time of physics. To avoid causality contradictions time travel into the past is banned, the tilt angle of the time axes in the Epstein diagram ³ is always less than or equal to 90° .

In the formulation of entropy, the second law reads as follows:

In spontaneously occurring processes $\Delta S \geq 0$ is always true. For all irreversible processes entropy increases, for reversible ones it remains constant.

What would change about this statement, if it were true that $T' = T$ and $S' = S \cdot \sqrt{v}$? Nothing, since $\Delta S \geq 0$ is equivalent to $\Delta S \cdot \sqrt{v} \geq 0$ which is the same as $\Delta S' \geq 0$.

The positive square root term changes nothing on the inequality. The second law and the arrow of time are not jeopardized by the transformation of entropy.



"He denies the big bang theory!"

22 Interpretation and Meaning of Temperature

The decision as to whether we should leave entropy or temperature invariant, depends on how we want to understand temperature. One could also claim that this decision deepens the definition of temperature. The two possibilities are

- temperature is defined first and foremost through the *internal energy*, which is ascribed to a certain amount of material and usually measured only indirectly. In this case it must also transform as U does, that is, by multiplying by the root. However, in the melting process internal energy increases without any change in temperature !

or

- temperature is directly coupled with certain equilibrium states (in particular the triple points of the substances or the state of matter at standard pressure). The melting point of pure gold at standard pressure is defined by the fact that liquid gold begins to solidify or that solid nuggets of gold begin to liquify; that the kinetic energy of the individual atoms begins to compete with their binding energy in the metal. Then temperature makes a statement about the *ratio* that different forms of energy in a certain amount of material have to each other. Temperature then is invariant in the sense of STR.

The *energy* of a particle *is* dependent on the relative velocity of the observer, and therefore in almost all of the proposed approaches heat is not invariant. If we tend to the first assumption, we must accordingly transform temperature by multiplying by the root factor and leave the entropy invariant. We have no other option. All those who divide heat and temperature by the root, are definitely barking up the wrong tree. It would require some work to show exactly where the decisive mistake is in their derivations. In the first approach triple points and melting points are dependent on relative velocities, while the fact that two or three phases coexist, of course, is not dependent on the speed of the observer.

In contrast, the second approach combines temperature with certain equilibrium conditions which are either present or not present, regardless of the relative speed of an observer. If at standard pressure pieces of aluminum are swimming around in a melting batch of aluminum, the soup has a temperature of 933.473 K. Temperature obtains a significance that is absolute in the relativistic sense. Critical temperatures of superconductors, etc. retain their value, regardless of the reference system, because the concept of temperature is directly related to the internal structure of materials. Indeed, temperature initially *is* defined in this way, and by means of so-called fixed-point cells precisely calibrated reference temperatures are spread throughout the world (see section 7!).

Only the second approach establishes temperature as a fundamental unit of physics. We want to take this view as the basis of the following sections.

Avramov claims that astronomical observations speak against an increase and also against a decrease in temperature with increasing relative velocity. According to his view distant galaxies should appear either very hot, or should no longer be observable because of their low temperature. The argument suffers, however, from the fact that the cosmic redshift is not the result of relative motion. So-called cosmic 'jets', gas or plasma clouds which are ejected from very young stars with very high relativistic velocities perpendicular to the accretion disc would perhaps provide better quantitative evidence.

23 The Transformation of Temperature

We assume in the remaining sections of this work that temperature is a key variable which is directly connected to many structural properties of materials, and therefore is independent of the velocity of an observer. Thus

$$T' = T$$

In the following we will examine, what conclusions can be drawn from this approach.

24 The Transformation of k and R

From sections 22 and 8 the transformations for k and R are

$$k' = k \cdot \sqrt{\quad} \quad \text{and} \quad R' = R \cdot \sqrt{\quad}$$

The two gas constants are thus not cut from the same cloth as the speed of light c and the Planck constant h . It is therefore not appropriate to set the value of k to 1, as is done with systems of natural units like with c , \hbar and the gravitational constant G .

25 The Transformation of Physical Entropy

$S = k \cdot \ln(\Omega)$ applies for physical entropies, where Ω is the inverse of an invariant probability. Therefore entropy transforms the same as the Boltzmann constant k :

$$S' = S \cdot \sqrt{\quad}$$

Entropies of information theory however are relativistically invariant.

26 The Transformation of Molar Heat Capacities

From the definition

$$\Delta Q = n \cdot C_V \cdot \Delta T$$

we infer that C_V transforms like ΔQ , ΔU , k and R . We would arrive at the same result by way of statistics. For an ideal gas with f degrees of freedom of motion for the particles, the formula

$$C_V = \frac{1}{2} \cdot f \cdot R$$

applies. Therefore C_V must transform like R . The same applies to molar heat of fusion and molar heat of evaporation.

27 Summary

If one defines temperature as a fundamental unit of physics, as in sections 21 and 22 of this paper, then the following values are relativistically invariant:

- the temperature T (and temperature changes)
- the pressure P (and pressure changes)
- particle number N and amount of substance $n = N / N_A$
- thermodynamic efficiency
- kappa, i.e., the ratio of C_P and C_V

The following values are transformed by multiplying by the root factor:

- the volume V (and volume changes)
- the Boltzmann constant k and the universal gas constant R
- the average velocity of the particles relative to the center of mass
- the average energy of individual particles
- heat quantity Q , internal energy U , work W (and their changes)
- entropy S and enthalpy H (and their changes)
- the molar heat capacity C_P and C_V
- mole-related melting and evaporation energies

Taking into account these transformations, all the essential relationships of thermodynamics remain valid. The following relationships are form invariant:

- the first law of thermodynamics, thus $\Delta Q = \Delta U - \Delta W$
- the second law of thermodynamics, thus $\Delta S \geq 0$ in a closed system
- for ideal gases $C_P - C_V = R$ and $C_V = f \cdot R / 2$
- for ideal gases $P \cdot V = n \cdot R \cdot T = N \cdot k \cdot T$
- for non-relativistic ideal gases $\overline{E_{kin}} = \frac{3}{2} \cdot k \cdot T$
- for non-relativistic ideal gases $P = \frac{1}{3} \cdot \frac{N}{V} \cdot m \cdot \overline{u^2} = \frac{2}{3} \cdot \frac{N}{V} \cdot \overline{E_{kin}}$
- the enthalpy is $H = U + P \cdot V$
- the entropy is $dS \cdot T = dQ$ and $S = k \cdot \ln(\Omega)$ where Ω is the inverse of an invariant probability

It should be noted that we have not *assumed* the validity of these relationships for a fast-moving observer in order to derive the transformation rules!

We only assumed that such transformations exist, and in addition we have defined temperature within the limits of $k' \cdot T' = k \cdot T \cdot \sqrt{}$ so that it is relativistically invariant. Many of the results were established even before this decision (see section 15).

Planck, Einstein, Hasenöhl and von Mosengeil made no errors (unlike many others) in the derivation of their results. They just decided that $k' = k$ should apply, and then they derived their other results in a logically correct way.

Avramov, however, made the better choice!

28 The Proportion of Heat Energy to Total Energy

What is the contribution of heat energy to the total energy of a volume of gas ? Does the mass of a gas cloud noticeably increase when it is heated ?

We calculate this contribution for 1 mol of helium at a temperature of 100 million K. In the rest system of the gas cloud, we determine the proportions as follows:

- the mass at 0 K is $E_0 = m \cdot c^2 \approx 0.004 \text{ kg} \cdot c^2 \approx 3.59 \cdot 10^{14} \text{ J}$
- the internal energy is $U = N \cdot 1.5 \cdot k \cdot T \approx N_A \cdot 1.5 \cdot k \cdot 10^8 \text{ K} \approx 12.5 \cdot 10^8 \text{ J}$
- the 'external' energy is $P \cdot V = N \cdot k \cdot T \approx N_A \cdot k \cdot 10^8 \text{ K} \approx 8.3 \cdot 10^8 \text{ J}$

The total enthalpy $H = U + P \cdot V$ for a temperature of 100 million K accounts for less than a thousandth of one percent of the rest energy due to mass ! And this rest energy precisely consists of $E_0 + H$, heat and pressure likewise contribute to the inertia of the ensemble. However, the rest energy due to heat and pressure is negligible.

The total energy for a fast-moving observer consists of the rest energy and the kinetic energy. Transformation of total energy takes place according to the formula

$$E_{tot}' = E_{tot} / \sqrt{}$$

That may be the reason that some authors in general have energy contributions increase with the reciprocal of the root. But the transformation of total energy does in no way show how the individual contributions to the heat content have to be transformed. This will be worked out in chapter 29 .

29 Transformation of Total Energy E_{tot} of a given Quantity of an Ideal Gas

In the rest frame, a given quantity of an Ideal Gas represents an 'internal energy' of

$$E = M \cdot c^2 + U$$

M stands for the rest mass of the gas at 0 K. The thermal energy U adds a certain amount to the inertia of the gas. The calculation in section 28 has shown this additional mass to be very small even for high temperatures. For the total energy of the gas the 'external' energy $P \cdot V$ has to be added, and so we have

$$E_{tot} = E + P \cdot V = M \cdot c^2 + U + P \cdot V = M \cdot c^2 + H$$

So much to the situation in the rest frame.

For a fast moving observer the total energy of the gas has **two additional terms** both for kinetic energy. The first additional term is the well-known expression

$$E_{kin,1} = E \cdot (1/\sqrt{1 - v^2/c^2} - 1)$$

Following Pauli in his famous Encyclopedia paper of 1920 (section 43, p.684) we have to add a second additional term reflecting the *transport of the tension* in the container holding the gas in the volume V at pressure P . Pauli calculates this term as

$$E_{kin,2} = (v^2/c^2) \cdot P \cdot V / \sqrt{1 - v^2/c^2} = (v^2/c^2) \cdot P' \cdot V' / (1 - v^2/c^2)$$

So we get for the total energy of the gas for a fast moving observer the expression

$$E_{tot}' = E + E_{kin,1} + E_{kin,2} + P' \cdot V' = E/\sqrt{1 - v^2/c^2} + E_{kin,2} + P' \cdot V'$$

Some algebraic manipulation shows that the following holds:

$$E_{kin,2} + P' \cdot V' = (P \cdot V) / \sqrt{1 - v^2/c^2}$$

Thus we get the simple result

$$E_{tot}' = E/\sqrt{1 - v^2/c^2} + (P \cdot V) / \sqrt{1 - v^2/c^2} = (E + P \cdot V) / \sqrt{1 - v^2/c^2} = E_{tot} / \sqrt{1 - v^2/c^2}$$

This is the equation presented at the end of section 28. Together with Pauli we have given a proof of it :-)

If E' stands for the total internal energy, that is total energy minus the external energy $P' \cdot V'$, then the following lines are correct:

$$E_{tot} = E + P \cdot V \quad \text{and} \quad E_{tot}' = E' + P' \cdot V'$$

Furthermore, we have

$$E' + P' \cdot V' = (E + P \cdot V) / \sqrt{1 - v^2/c^2} \quad \text{where} \quad P' \cdot V' = P \cdot V \cdot \sqrt{1 - v^2/c^2} \quad !!$$

Using the notation of this section, it is possible to write an elegant expression for the *momentum* of a fast-moving gas. These results earn their own section 30.

By the way: the considerations in this and the following section are not affected whether we opt in favour of temperature or of entropy as a relativistic invariant !

30 Total Momentum \mathbf{p} of a fast moving Quantity of Gas

Momentum \mathbf{p} of a fast moving quantity of gas points in the same direction as the relative velocity \mathbf{v} . Pauli derives in his Encyclopedia paper of 1920 (section 43, p.648) the following equation :

$$\mathbf{p} = (E_{tot}' / c^2) \cdot \mathbf{v} = (E' + P' \cdot V') \cdot (1 / c^2) \cdot \mathbf{v}$$

Using the results of section 29 we get the equivalent formulation

$$\mathbf{p} = E_{tot} / (c^2 \cdot \gamma) \cdot \mathbf{v} = (E + P \cdot V) / (c^2 \cdot \gamma) \cdot \mathbf{v}$$

Formally, these expressions are completely identical to those corresponding to a mass point. Remembering the details of section 29 we learn that the thermal Energy U and the external forces holding the gas in the volume V (i.e. the thermodynamic work $P \cdot V$) both contribute to the total momentum of our gas.

The energy-momentum-4-vector can be calculated as usual as the product of E_{tot} / c^2 by the 4-velocity. And section 29 gives us all the details about the constituents of E_{tot} and E_{tot}' .

31 The Laws of Black Body Radiation just confirm the Transformation of $k \cdot T$

A cavity in a black body of temperature T is filled with radiation. Friedrich Hasenöhl has analyzed the state variables of this radiation (in thermal equilibrium) *and* the transformation of these state variables due to a velocity v of the cavity relative to an observer. He did this in 1904, that is before Einstein's first publication on STR !

Hasenöhl used nothing but electrodynamics (cf. Pauli's above mentioned publication, section 49, p.697). Radiation, moved in a box, carries momentum and mass ! Kurd von Mosengeil reformulated these results 1906 in terms of STR. Mosengeil died very young in a climbing accident. So Planck told another assistant of his, namely Max von Laue, to give von Mosengeil's dissertation the final touch. The complete results were published in 1907.

In all of these results a factor $a \cdot T^4$ shows up - the only exception is the entropy term. Now, the konstant a contains the Boltzmann value k to the power of 4, too (cf. Pauli's above mentioned publication, section 49, p.698)! Thus we only have a further path leading to the insight that the product $k \cdot T$ transforms for a fast moving observer by multiplying with the root. There is no additional hint on which of the state variables, temperature T or entropy S , should be invariant in STR.

32 Conditions and Limitations of this Work

- we assumed in section 2, that there should be scalar transformations for each of the thermodynamic state variables. This assumption appears justified in retrospect, since we have found two self-consistent sets of such transformations !
- based on our consideration of the behavior of ideal gases we derived the transformation of the product $k \cdot T$. In the interim balance of section 15 quite a lot of results were by necessity derived that were eventually compiled in section 27. However, these results must be applied with caution to real gases! The center of the sun with a temperature of 15.6 million K is well within our limits, but we no longer have a gas of neutral particles, but a plasma. Even if the particle velocities are still moderate the forces between the particles cannot be neglected.
- the equations for an ideal gas (but not our transformation formulas !!) cease to be valid at low temperatures, if the pressure in the gas is too high. When the free path length is no longer significantly larger than the particle diameter, the preconditions for some statistical considerations no longer apply.
- we make no claims about the relationship between temperature and average kinetic energy in a gas in which the particles move with relativistic velocities, i.e. faster than 10 or 20 % of c . But this changes nothing about the transformations of the variables P , V , T , S , N , U and H !
- if one defines temperature like Avramov in the meaning of sections 21 and 22, then the constants k and R must transform by multiplying by the root factor. This is in no way restricted to the realm of ideal gases.
- it is possible to interpret entropy S as a relativistic invariant. Then the Boltzmann constant is invariant, and the temperature T must transform by multiplying by the root factor: 'Fast-moving objects appear cooler'. It is doubtful, however, that this version is compatible with astronomical observations. This would not influence the transformations of P , V , N , U and H .
- the zero point of the absolute temperature scale is in any case relativistically invariant. The position of the zero point does not depend on whether T is invariant or is multiplied by the root factor.
- today or in the near future, it is quite unlikely that these transformations will find a practical application in a terrestrial laboratory. How would one investigate a large hot ensemble of gas under controlled conditions at $v = 0.8 \cdot c$?
- however, astronomers observe very hot gases that move at high relativistic speeds (so-called 'jets', see the wikipedia contributions to astronomical (cosmic, or polar) jets and the keyword 'superluminal motion'). How do astronomers determine the temperature of such objects? What relativistic corrections do they apply?

33 A Very Incomplete Review of the Literature

- all authors agree on how to transform the volume
- all authors (except Popovic 2008 ...) agree on how to transform pressure
- we agree with Einstein, Planck and Landsberg on how to transform quantities of heat, while Eddington, Blausa and Ott write the root factor in the denominator
- for none of these renowned authors the temperature is invariant. It cannot be, since they all assume that k and hence the entropy S are invariant
- for Einstein and Planck, the temperature decreases by the root factor with increasing relativistic velocities while with Eddington, Ott and some others it correspondingly increases
- in 1969 van Kampen stated that the measurement of a temperature of a fast-moving object is not possible, and, therefore, temperatures cannot be sensibly transformed within the STR
- Landsberg claimed in 1970, that temperature *and* entropy are invariant. Avramov said in 2003 that this was the first attempt to set temperature as an invariant value
- in 1996 Landsberg and Matsas claim that in the STR no consistent transformation can be specified for the temperature
- Avramov in 2003 makes a good case for the claim that the temperature is invariant, and he correctly concludes that k and the entropy S must transform
-> <http://www.ipc.bas.bg/PPages/Avramov/lectReliv.pdf>
- in 2006 Khaleghy and Qassemi again argued that temperatures would increase with increasing relativistic velocity
- in 2007 Cubero and Hänggi do a computer simulation of a one-dimensional (!) gas and come to the conclusion that temperature is an invariant value
-> <http://www.pro-physik.de/details/news/prophy9817news/news.html?laid=9817>
- Popovic touted in 2008 that $P = P \cdot \gamma$ (along with $T' = T$ and $S' = S$) (should be ignored ...)
- in 2008 Requard attacks with the entire arsenal of tensor calculus. He assumes without reflection that $S' = S$, and ends with the results of Eddington and Ott
- Gérard P. Michon, on his website (2000 - 2013) , also assumes $S' = S$ and, logically correct, comes to the same results as Einstein and Planck ('von Mosengeil's formula').
-> www.numericana.com/answer/heat.htm#relativistic
- finally in 2013 a unknown physics teacher writes this review article, attempting thereby, to help the concept of Avramov and Cuberto/Hänggi to a breakthrough ...
- at least a dozen other publications have thrown no new light on the situation